MODEL ANSWERS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 20 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3C**

Further Mathematics

Advanced

Paper 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1.

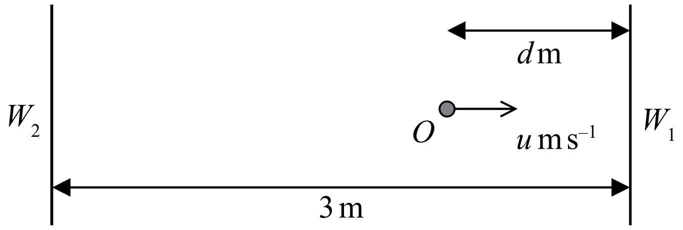


Figure 1

Figure 1 represents the plan of part of a smooth horizontal floor, where W_1 and W_2 are two fixed parallel vertical walls. The walls are 3 metres apart.

A particle lies at rest at a point O on the floor between the two walls, where the point O is d metres, $0 < d \leqslant 3$, from W_1

At time $t = 0$, the particle is projected from O towards W_1 with speed $u\,\text{m s}^{-1}$ in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$ $\rightarrow e = \frac{2}{3}$

The particle returns to O at time $t = T$ seconds, having bounced off each wall once.

(a) Show that $T = \dfrac{45 - 5d}{4u}$

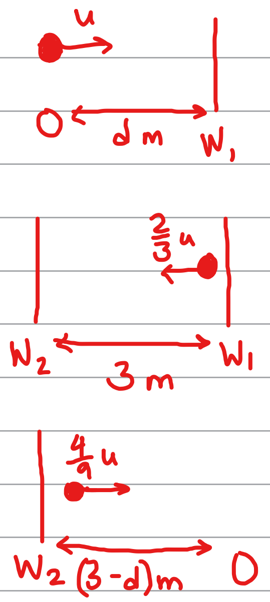
(6)

The value of u is fixed, the particle still hits each wall once but the value of d can now vary.

(b) Find the least possible value of T, giving your answer in terms of u. You must give a reason for your answer.

(2)

(a)



From O to W_1: Speed after 1st impact, $v_1 = eu = \frac{2}{3}u$

Time after 1st impact, $t_1 = \frac{d}{u}$

From W_1 to W_2: Speed after 2nd impact, $v_2 = e^2u = \frac{4}{9}u$

Time after 2nd impact, $t_2 = \dfrac{3}{\frac{2}{3}u} = \dfrac{9}{2u}$ $\times 3$

From W_2 to O:

Remaining time, $t_3 = \dfrac{3-d}{\frac{4}{9}u} = \dfrac{27-9d}{4u}$ $\times 9$

$T = t_1 + t_2 + t_3 = \dfrac{d}{u} + \dfrac{9}{2u} + \dfrac{27-9d}{4u} = \dfrac{4d + 18 + 27 - 9d}{4u} = \boxed{\dfrac{45-5d}{4u}}$ ($\times 4$, $\times 2$)

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Question 1 continued

(b.) T has the least possible value when d is maximum.

$\therefore d = 3 \quad \therefore$ Least $T = \dfrac{45 - 5(3)}{4u} = \dfrac{30}{4u} = \boxed{\dfrac{15}{2u}}$

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2.

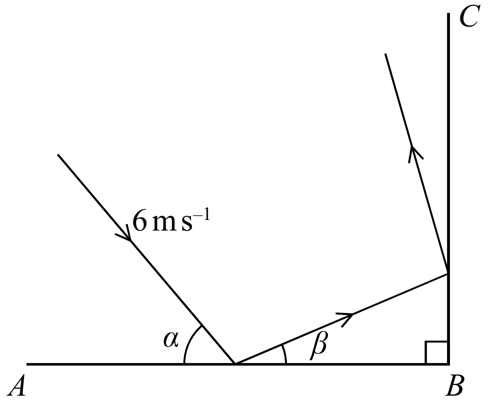


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where AB and BC are fixed vertical walls with AB perpendicular to BC.

A small ball is projected along the floor towards AB with speed $6\,\mathrm{m\,s^{-1}}$ on a path that makes an angle α with AB, where $\tan\alpha = \frac{4}{3}$. The ball hits AB and then hits BC. Immediately after hitting AB, the ball is moving at an angle β to AB, where $\tan\beta = \frac{1}{3}$

The coefficient of restitution between the ball and AB is e. $\rightarrow e_{AB} = e$

The coefficient of restitution between the ball and BC is $\frac{1}{2}$ $\rightarrow e_{BC} = \frac{1}{2}$

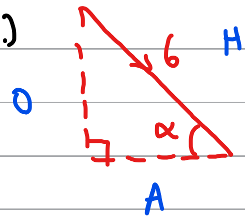
By modelling the ball as a particle and the floor and walls as being smooth,

(a) show that the value of $e = \frac{1}{4}$
(5)

(b) find the speed of the ball immediately after it hits BC.
(4)

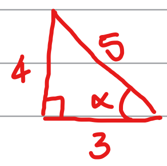
(c) Suggest two ways in which the model could be refined to make it more realistic.
(2)

(a.)

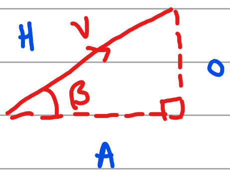


Parallel Component: $6\cos\alpha$
Perpendicular Component: $6\sin\alpha$

$\tan\alpha = \frac{4}{3}$

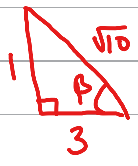


$\therefore \cos\alpha = 3/5$
$\therefore \sin\alpha = 4/5$



Parallel component: $v\cos\beta$
Perpendicular component: $v\sin\beta$

$\tan\beta = \frac{1}{3}$



$\therefore \cos\beta = 3/\sqrt{10}$
$\therefore \sin\beta = 1/\sqrt{10}$

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Question 2 continued

$\leftrightarrow$: $u\cos\alpha = v\cos\beta$

$\therefore 6\cos\alpha = v\cos\beta$

$6\left(\frac{3}{5}\right) = v\left(\frac{3}{\sqrt{10}}\right) \Rightarrow \therefore v = \dfrac{6\left(\frac{3}{5}\right)}{\left(\frac{3}{\sqrt{10}}\right)} = \dfrac{6\sqrt{10}}{5}$

$\updownarrow$: $eu\sin\alpha = v\sin\beta$

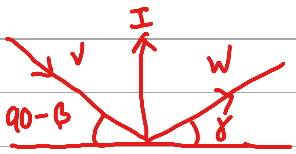
$\therefore 6e\sin\alpha = v\sin\beta$

$6e\left(\frac{4}{5}\right) = \left(\frac{6\sqrt{10}}{5}\right)\left(\frac{1}{\sqrt{10}}\right)$

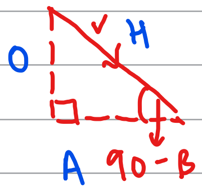
$\frac{24}{5}e = \frac{6}{5} \Rightarrow \boxed{\therefore e = \frac{6}{24} = \frac{1}{4}}$

$\times 5 \quad \times 5$

(b.)

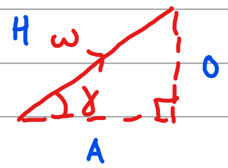


$v = \dfrac{6\sqrt{10}}{5}$



Parallel Component: $v\cos(90-\beta) = \frac{6\sqrt{10}}{5}\sin\beta = \frac{6\sqrt{10}}{5} \times \frac{1}{\sqrt{10}} = \frac{6}{5}$

Perpendicular Comp.: $v\sin(90-\beta) = \frac{6\sqrt{10}}{5}\cos\beta = \frac{6\sqrt{10}}{5} \times \frac{3}{\sqrt{10}} = \frac{18}{5}$



Parallel component : $w\cos\gamma$

Perpendicular Component : $w\sin\gamma$

$u\cos\alpha = v\cos\beta \Rightarrow v\cos(90-\beta) = w\cos\gamma$

$\therefore w\cos\gamma = \frac{6}{5}$

$eu\sin\alpha = v\sin\beta \Rightarrow ev\sin(90-\beta) = w\sin\gamma$

$\frac{1}{2} \times \frac{18}{5} = w\sin\gamma$

$\therefore w\sin\gamma = \frac{9}{5}$

Squaring both equations.

$w^2\cos^2\gamma + w^2\sin^2\gamma = \left(\frac{6}{5}\right)^2 + \left(\frac{9}{5}\right)^2$

$w^2(\cos^2\gamma + \sin^2\gamma) = \frac{117}{25} \Rightarrow \boxed{\therefore w = \sqrt{\frac{117}{25}} = \frac{3\sqrt{13}}{5}\ \text{ms}^{-1}}$

Question 2 continued

(c.) Refinements:

① Include friction between floor and ball.
② Include friction between ball and walls.
③ Consider dimensions of ball.
④ Consider rotational effects of ball.
⑤ Consider air resistance.

Question 2 continued

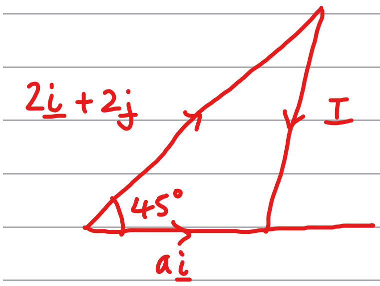
(Total for Question 2 is 11 marks)

3. A particle P, of mass 0.5 kg, is moving with velocity $(4\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{I}$ of magnitude 2.5 Ns.

As a result of the impulse, the direction of motion of P is deflected through an angle of 45°

Given that $\mathbf{I} = (\lambda\mathbf{i} + \mu\mathbf{j})$ Ns, find all the possible pairs of values of λ and μ.

(9)



P: 0.5 kg , $|I| = 2.5$ Ns

Momentum of P after impulse $= a\underline{i}$

$I = m(v-u)$

$I = 0.5(a\underline{i} - 4\underline{i} + a\underline{j} - 4\underline{j})$

$= 0.5(2a\underline{i} - 4\underline{i} - 4\underline{j})$

$= a\underline{i} - 2\underline{i} - 2\underline{j}$

$= (a-2)\underline{i} - 2\underline{j}$

$|I| = 2.5 = \sqrt{\lambda^2 + \mu^2}$

$\therefore 2.5^2 = 6.25 = \sqrt{\lambda^2+\mu^2}$

$6.25 = (a-2)^2 + (-2)^2$

$\frac{25}{4} = a^2 - 4a + 4 + 4$

$25 = 4a^2 - 16a + 32$

$\therefore 4a^2 - 16a + 7 = 0$

$(2a-1)(2a-7) = 0$

$\therefore a = \frac{1}{2}, \; a = \frac{7}{2}$

$\therefore I = \left(\frac{7}{2} - 2\right)\underline{i} - 2\underline{j} = \frac{3}{2}\underline{i} - 2\underline{j}$

$\therefore I = \left(\frac{1}{2} - 2\right)\underline{i} - 2\underline{j} = -\frac{3}{2}\underline{i} - 2\underline{j}$

$\therefore I = -2\underline{i} + \left(\frac{7}{2} - 2\right)\underline{j} = -2\underline{i} + \frac{3}{2}\underline{j}$

$\therefore I = -2\underline{i} + \left(\frac{1}{2} - 2\right)\underline{j} = -2\underline{i} - \frac{3}{2}\underline{j}$

$$\boxed{\begin{array}{l} \therefore \lambda = \frac{3}{2}, \mu = -2 \;\& \\ \lambda = -\frac{3}{2}, \mu = -2 \;\& \\ \lambda = -2, \mu = \frac{3}{2} \;\& \\ \lambda = -2, \mu = -\frac{3}{2} \end{array}}$$

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Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4. A car of mass 600 kg pulls a trailer of mass 150 kg along a straight horizontal road. The trailer is connected to the car by a light inextensible towbar, which is parallel to the direction of motion of the car. The resistance to the motion of the trailer is modelled as a constant force of magnitude 200 N. At the instant when the speed of the car is $v\,\mathrm{m\,s^{-1}}$, the resistance to the motion of the car is modelled as a force of magnitude $(200 + \lambda v)$ N, where λ is a constant.

When the engine of the car is working at a constant rate of 15 kW, the car is moving at a constant speed of $25\,\mathrm{m\,s^{-1}}$

(a) Show that $\lambda = 8$

(4)

Later on, the car is pulling the trailer up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{15}$

The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 200 N at all times. At the instant when the speed of the car is $v\,\mathrm{m\,s^{-1}}$, the resistance to the motion of the car from non-gravitational forces is modelled as a force of magnitude $(200 + 8v)$ N.

The engine of the car is again working at a constant rate of 15 kW.

When $v = 10$, the towbar breaks. The trailer comes to instantaneous rest after moving a distance d metres up the road from the point where the towbar broke.

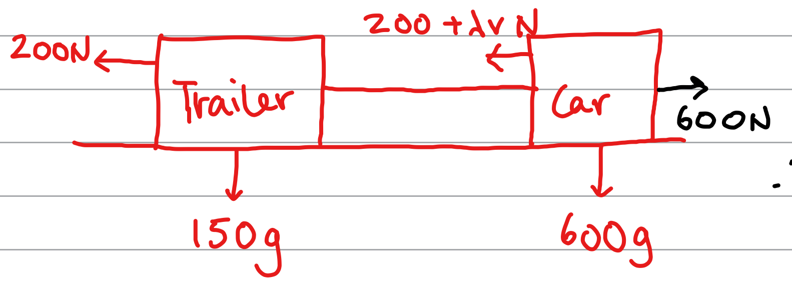
(b) Find the acceleration of the car immediately after the towbar breaks.

(4)

(c) Use the work-energy principle to find the value of d.

(4)

(a) $\rightarrow v = 25\,\mathrm{ms^{-1}}$



$P = D_f v = 15\,\mathrm{kW}$

$\therefore D_f = \dfrac{15000}{25} = 600\,\mathrm{N}$

$25\,\mathrm{ms^{-1}}$ is constant speed $\therefore$ Driving Forces = Resistive Forces

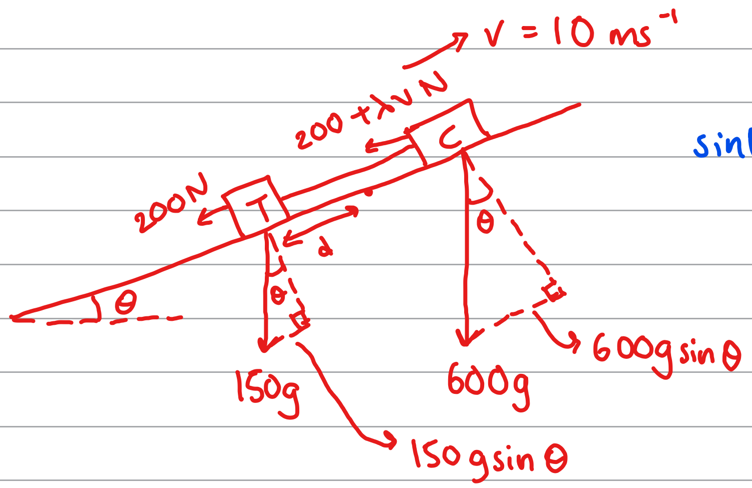
$600 = 200 + 200 + 25\lambda$

$600 = 400 + 25\lambda$

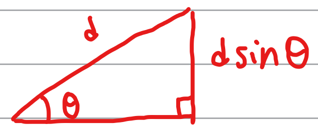
$\therefore \lambda = \dfrac{600 - 400}{25} = 8$ $\quad \boxed{\therefore \lambda = 8}$

Question 4 continued

(b.)



$\sin\theta = \frac{1}{15}$, $\cos\theta = \frac{4\sqrt{14}}{15}$



$P = 15\text{kW}$, $v = 10\ \text{ms}^{-1}$

$F = ma \Rightarrow \frac{15000}{10} - (200 + 8(10)) - 600g\sin\theta = 600a$

$1500 - 280 - 600(9.8)\left(\frac{1}{15}\right) = 600a$

$1500 - 280 - 392 = 600a$

$828 = 600a$

$a = \frac{828}{600} = 1.38$

$\boxed{\therefore a = 1.38\ \text{ms}^{-2}}$

(c.) Energy at start + Work Done by Driving Forces = Energy at End + Work Done by Resistive Forces

KE of trailer = GPE gained by Trailer + WD by Resistance

$\frac{1}{2}(150)(10)^2 = (150)(9.8)(d\sin\theta) + 200d$

$7500 = \frac{1470d}{15} + 200d$

$7500 = 98d + 200d$

$7500 = 298d \quad \therefore d = \frac{7500}{298} = 25.16\ldots \approx \boxed{25.2\text{ m}}$ (3sf)

Question 4 continued

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Question 4 continued

(Total for Question 4 is 12 marks)

5. A particle P of mass $3m$ and a particle Q of mass $2m$ are moving along the same straight line on a smooth horizontal plane. The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is u and the speed of Q is $2u$.

Immediately after the collision P and Q are moving in opposite directions.

The coefficient of restitution between P and Q is e.

(a) Find the range of possible values of e, justifying your answer.

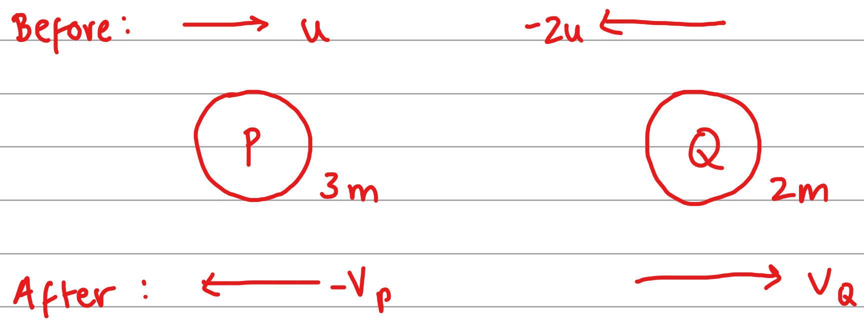
(8)

Given that Q loses 75% of its kinetic energy as a result of the collision,

(b) find the value of e.

(3)

(a)



$p = mv$

$v = eu$

$e = \dfrac{v_B - v_A}{u_A - u_B}$

Conservation of Momentum : $3mu + 2m(-2u) = -3mV_P + 2mV_Q$

$3u - 4u = -3V_P + 2V_Q$

$\therefore -3V_P + 2V_Q = -u$ (1)

Impact Law : $e = \dfrac{V_Q - -V_P}{u - -2u}$

$\therefore 3ue = V_P + V_Q$ (2)

Solving simultaneous equations : (1) : $-\cancel{3V_P} + 2V_Q = -u$

$+$ (2) $\times 3$: $\cancel{3V_P} + 3V_Q = 9ue$

$5V_Q = 9ue - u$

$\therefore V_Q = \dfrac{9ue - u}{5}$

$V_Q > 0$, $\therefore \dfrac{9ue - u}{5} > 0 \Rightarrow 9ue - u > 0$

$u(9e - 1) > 0$

$9e - 1 > 0$

$\therefore e > \dfrac{1}{9}$

$\boxed{\text{As } 0 \leq e \leq 1, \quad \therefore \frac{1}{9} < e \leq 1.}$

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Question 5 continued

(b.) Q lose 75% KE.

$\therefore$ Final KE = 25% of Initial KE.

$KE = \frac{1}{2}mv^2$

$\frac{1}{2}(2m)\left(\frac{9eu-u}{5}\right)^2 = 0.25 \times \frac{1}{2}(2m)(-2u)^2$

$\frac{\cancel{mu^2}(9e-1)^2}{25} = \cancel{mu^2}$

$(9e-1)^2 = 25$

$9e-1 = \sqrt{25}$

$9e-1 = 5$

$e = \frac{5+1}{9} = \frac{6}{9} = \frac{2}{3}$ $\boxed{\therefore e = \frac{2}{3}}$

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A smooth uniform sphere A has mass 0.2 kg and another smooth uniform sphere B, with the same radius as A, has mass 0.4 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision, the velocity of A is $(3\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the velocity of B is $(-4\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$

At the instant of collision, the line joining the centres of the spheres is parallel to **i**

The coefficient of restitution between the spheres is $\frac{3}{7}$ $\rightarrow e = \frac{3}{7}$

(a) Find the velocity of A immediately after the collision.

(7)

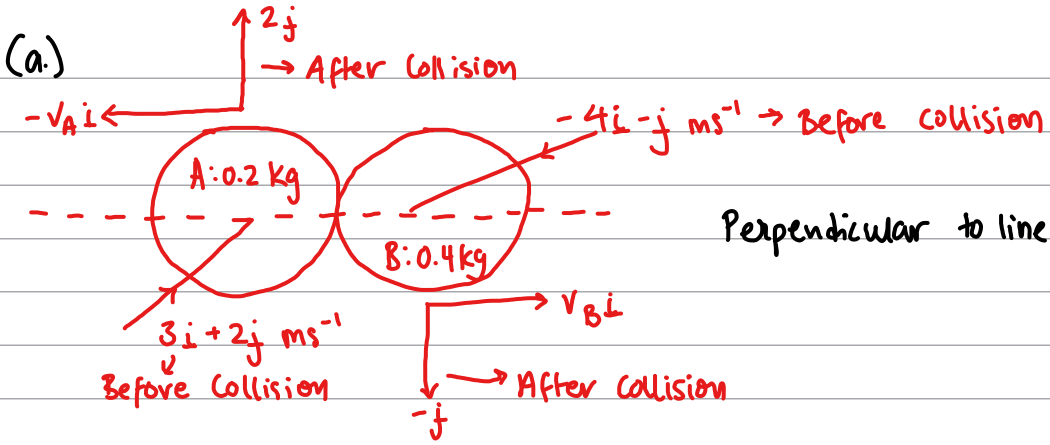
(b) Find the magnitude of the impulse received by A in the collision.

(2)

(c) Find, to the nearest degree, the size of the angle through which the direction of motion of A is deflected as a result of the collision.

(3)

(a.)



Perpendicular to line of centres: $2j$

Conservation of Linear Momentum (CLM): $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

CLM Parallel to line of centres: $0.2(3) + 0.4(-4) = 0.2(-v) + 0.4(w)$

$0.6 - 1.6 = -0.2v_A + 0.4v_B$

$-1 = -\frac{v_A}{5} + \frac{2v_B}{5}$

$\times 5 \quad \times 5 \quad \times 5$

$\therefore\ 2v_B - v_A = -5$ ①

Impact Law: $e(u_A - u_B) = v_B - v_A$

Impact law parallel to line of centres: $e(3 - -4) = v_B - -v_A$

$7\left(\frac{3}{7}\right) = v_B + v_A$

$\therefore\ v_B + v_A = 3$ ②

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Question 6 continued

Solving simultaneous equations: ① : $\cancel{2V_B} - V_A = -5$

$- ②\times 2 : \cancel{2V_B} + 2V_A = 6$

$-3V_A = -11$

$\therefore V_A = \frac{11}{3}$ → This is magnitude of velocity parallel to line of centres.

$\boxed{\therefore \text{Velocity of A after collision} = -\frac{11}{3}\mathbf{i} + 2\mathbf{j} \text{ ms}^{-1}}$

(b.) $|I| = |m(v-u)|$

$|I| = 0.2\left|-\frac{11}{3} - 3\right|$

$|I| = 0.2 \times \frac{20}{3}$

$\boxed{\therefore |I| = \frac{4}{3} \text{ Ns}}$

(c.) Scalar Product: $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A||B|}$, $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$

$$\cos\theta = \frac{(3\mathbf{i}+2\mathbf{j})\cdot(-\frac{11}{3}\mathbf{i}+2\mathbf{j})}{\sqrt{3^2+2^2} \times \sqrt{(\frac{11}{3})^2+2^2}}$$

$$\cos\theta = \frac{3(-\frac{11}{3}) + 2(2)}{\sqrt{13} \times \frac{\sqrt{157}}{3}} = \frac{-7}{\left(\frac{\sqrt{2041}}{3}\right)} = \frac{-21}{\sqrt{2041}}$$

$\theta = \cos^{-1}\left(\frac{-21}{\sqrt{2041}}\right) = 117.69\ldots \approx 118°$ (Nearest Degree)

$\boxed{\therefore \theta = 118°}$

Question 6 continued

Question 6 continued

(Total for Question 6 is 12 marks)

7. A particle P, of mass m, is attached to one end of a light elastic spring of natural length a and modulus of elasticity kmg.

The other end of the spring is attached to a fixed point O on a ceiling.

The point A is vertically below O such that $OA = 3a$

The point B is vertically below O such that $OB = \frac{1}{2}a$

The particle is held at rest at A, then released and first comes to instantaneous rest at the point B.

(a) Show that $k = \frac{4}{3}$

(3)

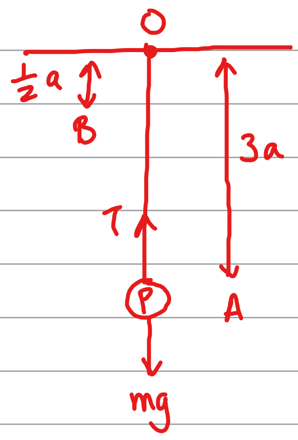
(b) Find, in terms of g, the acceleration of P immediately after it is released from rest at A.

(3)

(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B.

(6)

(a) $T = \frac{\lambda x}{l}$, $EPE = \frac{\lambda x^2}{2l}$ $\lambda = kmg$, $l = a$



Energy at start = Energy at End

EPE = GPE + EPE

$$\frac{kmg(3a-a)^2}{2a} = mg(3a - \tfrac{1}{2}a) + \frac{kmg(a-\frac{1}{2}a)^2}{2a}$$

$$\frac{4kmga^2}{2a} = \frac{5}{2}mga + \frac{\frac{1}{4}kmga^2}{2a}$$

$$2kmga = \frac{5}{2}mga + \frac{kmga}{8}$$

$\div mga$ $\div mga$ $\div mga$

$$2k = \frac{5}{2} + \frac{k}{8}$$

$$\left(2 - \frac{1}{8}\right)k = \frac{5}{2}$$

$$k = \frac{5}{2} \div \frac{15}{8} = \frac{4}{3}$$

$\boxed{\therefore k = \frac{4}{3}}$

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Question 7 continued

(b.) $F = ma$, $T = \frac{\lambda x}{l}$

$T - mg = mA$

$$\frac{\frac{4}{3}mg(3a-a)}{a} - mg = mA$$

$$\frac{4\cancel{m}g(2\cancel{a})}{3\cancel{a}} - \cancel{m}g = \cancel{m}A$$

$\div m$ $\div m$ $\div m$

$$\frac{8}{3}g - g = A$$

$$\boxed{\therefore A = \frac{5}{3}g \text{ ms}^{-2}}$$

(c.) Maximum Speed Occurs at Equilibrium Position.

At equilibrium position $T = mg$.

$$\frac{\frac{4}{3}\cancel{mg}x}{a} = \cancel{mg}$$

$\div mg$ $\div mg$

$$\frac{4x}{3a} = 1$$

$$\therefore \text{Extension}, x = \frac{3a}{4}$$

Conservation of Energy from A to Equilibrium Position:

EPE = KE + EPE + GPE

$$\frac{\frac{4}{3}\cancel{m}g(3a-a)^2}{2a} = \frac{1}{2}\cancel{m}v^2 + \frac{\frac{4}{3}\cancel{m}g\left(\frac{3}{4}a\right)^2}{2a} + \cancel{m}g\left(3a - \frac{3}{4}a - a\right)$$

$\div m$ $\div m$ $\div m$ $\div m$

$$\frac{2g(4a^2)}{3a} = \frac{1}{2}v^2 + \frac{2g\left(\frac{9}{16}a^2\right)}{3a} + \frac{5}{4}ga$$

$$\frac{8}{3}ga = \frac{1}{2}v^2 + \frac{3}{8}ga + \frac{5}{4}ga \Rightarrow v^2 = 2\left(\frac{8}{3} - \frac{3}{8} - \frac{5}{4}\right)ga$$

$$v^2 = \frac{25}{12}ga \Rightarrow \boxed{\therefore v = \frac{5}{2}\sqrt{\frac{ga}{3}} \text{ ms}^{-1}}$$

Question 7 continued

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS